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National Aeronautics and Space Administration  
Goddard Space Flight Center  
Contract No. NAS-5-9299

ST - PF - 10498

EFFECT OF RADIATION ATTENUATION UPON THE MOTION  
OF A RELATIVISTIC PARTICLE IN A  
UNIFORM MAGNETIC FIELD

by

G. E. Gernet

(USSR)

GPO PRICE \$ \_\_\_\_\_

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) 3.00

Microfiche (MF) 165

ff 853 July 65

FACILITY FORM 802	<b>N67 16607</b>	
	(ACCESSION NUMBER)	(THRU)
	<i>5</i>	<i>1</i>
	(PAGES)	(CODE)
	<i>CR-81270</i>	<i>34</i>
	(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

10 JUNE 1966

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Doklady A. N. SSSR  
 Fizika,  
 Tom 168, No. 2, 63-64,  
 Izdatel'stvo "NAUKA", 1966

by G. E. Gernet

SUMMARY

The effect of radiation decrease during the motion of a relativistic particle in a uniform magnetic field is obtained by deriving the equations for the transverse components and finding an approximate value of the proper time.

\* \* \*

When a charged particle moves in a magnetic field, its energy decreases on account of bremsstrahlung (synchrotron) radiation, which leads to modification of particle's trajectory.

In the cases when the time of particle sojourn in the field is comparable with the characteristic time of energy decrease

$$T \sim m^3 c^5 / e^4 H^2$$

(as, for example, for electrons in cosmic fields), it is necessary to take into account the radiation attenuation when determining the trajectory.

The particle's equation of motion has the form

$$\frac{dp}{dt} = \frac{e}{c} [\mathbf{v}, \mathbf{H}] + \mathbf{f}_r, \quad (1)$$

where the deceleration force  $\mathbf{f}_r$  is [1]

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\* VLIYANIYE RADIATIONNOGO TORMOZHENIYA NA DVIZHENIYE RELYATIVISTSKOY CHASTITSY V ODNORODNOM MAGNITNOM POLE

$$f_T = \frac{2e^4}{3m^2c^5} \left\{ [H [H, v]] - \frac{v}{1-v^2/c^2} \frac{1}{c^2} ([v, H])^2 \right\}. \quad (2)$$

Let us direct the axis  $z$  of the Descartes system of coordinates along the field and denote

$$\omega = eH / mc, \quad \delta = 2/3 e^4 H^2 / m^3 c^5. \quad (3)$$

We shall express the velocity  $\underline{v}$  in fractions of  $\underline{c}$  and the energy  $E$  in fractions of  $\underline{mc}^2$ :

$$u = v / c, \quad w = E / mc^2 = 1 / \sqrt{1 - u^2}.$$

Then, passing to components and taking into account  $p = Ev / c^2$ , we shall obtain

$$\begin{aligned} du_x w / dt &= \omega u_y - \delta u_x (1 - u_z^2) w^2, \\ du_y w / dt &= -\omega u_x - \delta u_y (1 - u_z^2) w^2, \\ du_z w / dt &= -\delta u_z w^2 (u_x^2 + u_y^2). \end{aligned} \quad (4)$$

From (4) it follows, first of all, that

$$u_z = \text{const.} \quad (5)$$

Taking into account (5), we obtain from (4) the equation for energy variation

$$dw / dt = -\delta [(w / w_\infty)^2 - 1], \quad (6)$$

where

$$w_\infty = 1 / \sqrt{1 - u_z^2} \quad (7)$$

is the limit value of  $w$  at  $t \rightarrow \infty$ .

The integral of (6) is

$$w = w_\infty \text{cth} (\delta t / w_\infty + C_0). \quad (8)$$

The constant  $C_0$  is determined from the condition  $w = w_0$  at  $t = 0$ .

It is appropriate to express the quantity  $w_\infty$  by the angle  $\Theta$  between the velocity's initial direction and the direction of the field and by the initial value of energy. Assuming in (7)  $u_z = u_0 \cos \Theta$ , and taking into account that  $w_0 = 1 / \sqrt{1 - u_0^2}$ , we have

$$w_\infty = \frac{w_0}{\sqrt{\cos^2 \Theta + w_0^2 \sin^2 \Theta}}. \quad (9)$$

Utilizing (8), it is possible to integrate the equations for the

transverse components  $u_x$  and  $u_y$

$$\begin{aligned} u_x &= u_{\perp}(0) e^{-\delta\tau} \sin(\omega\tau + \varphi_0), \\ u_y &= u_{\perp}(0) e^{-\delta\tau} \cos(\omega\tau + \varphi_0), \end{aligned} \quad (10)$$

where

$$u_{\perp}(0) = \sqrt{u_x^2(0) + u_y^2(0)};$$

$\varphi_0$  is the initial phase;  $\tau$  is the proper time.

$$\tau = \int_0^t \frac{dt}{w} = \frac{1}{\delta} \ln \frac{\text{ch}(\delta t/w_{\infty} + C_0)}{\text{ch} C_0}. \quad (11)$$

It may be seen from (10) that the transverse components of velocity damp and convert to 0 at  $t \rightarrow \infty$ .

The proper time is a complex function of  $t$ , and this is contrary to the case of motion without taking into account the radiation friction, when  $\tau = t/w_0$ . As a consequence of this, the variation of  $u_x$  and  $u_y$  with time will no longer be harmonic, which will exert influence on the emission spectrum.

The equations obtained are substantially simplified at passing to the extreme relativistic case characterized by the correlation  $w_0 \gg 1$ .

Beyond the narrow cone around the field direction with an angle  $\theta_0 \sim 1/w_0^2$  we have from (9)  $w_{\infty} \approx 1/\sin \theta$ . Then (8) passes to

$$\frac{1}{w} = \frac{1}{w_0} + \sin \theta \text{th}(\delta t \sin \theta). \quad (12)$$

Inasmuch as  $\text{th} x$  attains practically its limit value for a value of  $x$  a little greater than the unity, the principal energy loss takes place in the time

$$t_0 \sim 1/\delta \sin \theta. \quad (13)$$

In the  $0 \leq t \leq t_0$  interval it is sufficient to limit oneself to the first term of the expansion of  $\text{th} x$  in series, so that

$$\frac{1}{w} = \frac{1}{w_0} + (\sin^2 \theta) \delta t \quad (t \delta \sin \theta \ll 1). \quad (14)$$

Hence it may be seen that the energy decreases by a factor of 2 for a time  $\sim 1/w_0 \delta \sin^2 \theta$ .

For the proper time we have in the same approximation

$$\tau = \frac{t}{w_0} + t^3 \frac{\delta}{2} \sin^2 \theta. \quad (15)$$

Substituting (15) into (10), we shall obtain the approximate expressions for the transverse velocity components.

The author conveys his gratitude to Prof. L.E. Gurevich for discussing the work.

\*\*\*\* THE END \*\*\*\*

The Leningrad Institute of  
Radio Engineering and Communications  
in the name of M.A. Bonch-Bruyevich.

Manuscript  
received  
on 3 June 1965.

#### REFERENCE

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Contract No.NAS-5-9299  
Consultants and Designers, Inc.  
Arlington, Virginia

Translated by ANDRE L. BRICHANT  
on 9 June 1966

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